

Holographic Study on the Phase Structure of QCD Matter under Rotation

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Based on works :

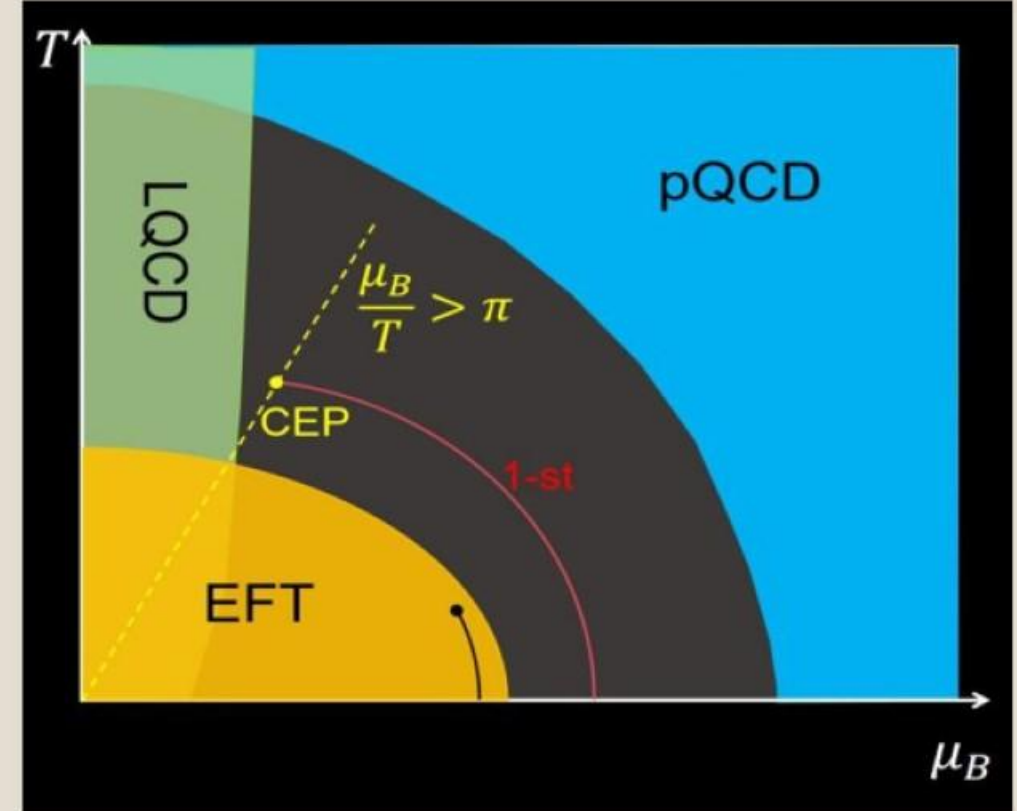
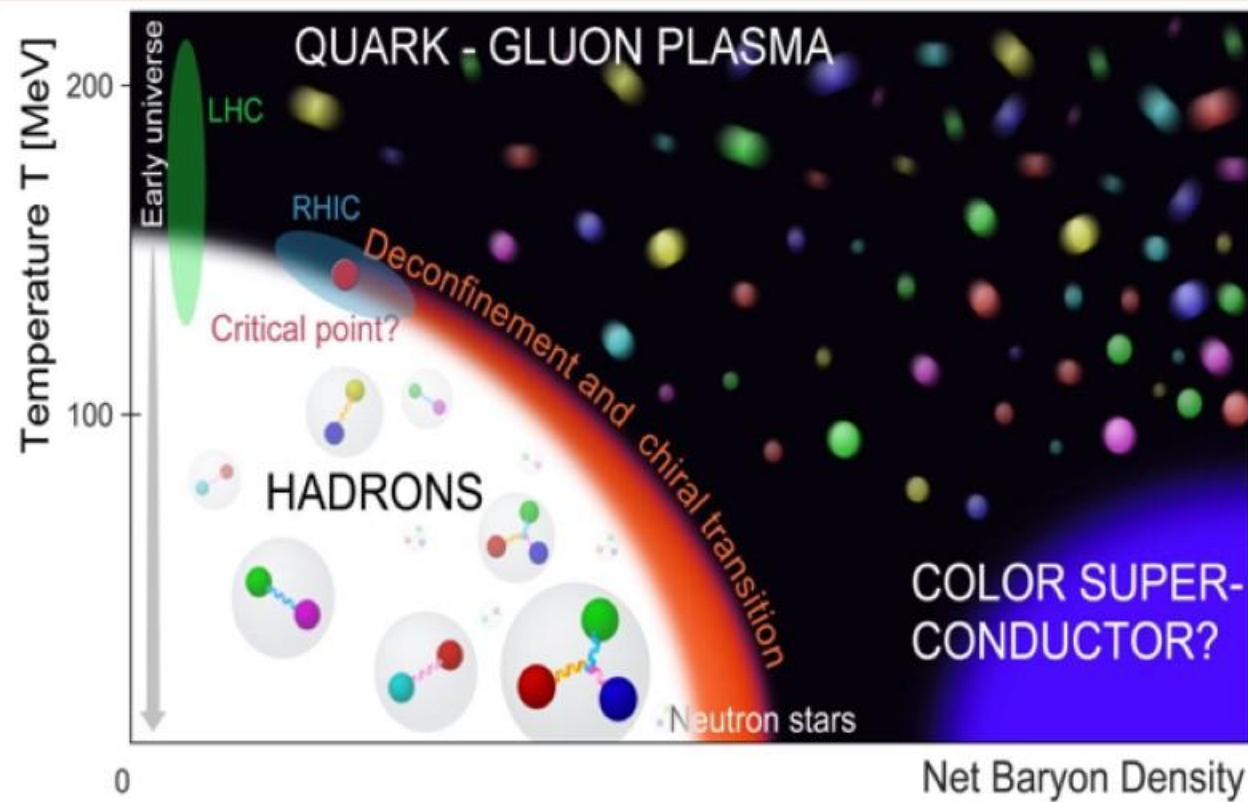
- Wang, Wei, **ZL**, Huang, [Phys.Rev.D 99 \(2019\) 1, 016018](#).
- Zhao, He, Hou, Li, **ZL**, [10.1007/JHEP04\(2023\)115](#).
- **ZL**, Fei Wang, [arXiv: 2507.09113](#).
- **ZL**, Yidian Chen, Mei Huang, [arXiv: 2607.23199](#).

The 2nd Workshop on QCD Matter under Rotation and Magnetic Fields, Huainan

Outline :

- 1. Motivation**
- 2. Holographic renormalization**
- 3. Pseudo-Lorentz boost method**
- 4. Rotation–magnetic correspondence**
- 5. Conclusion**

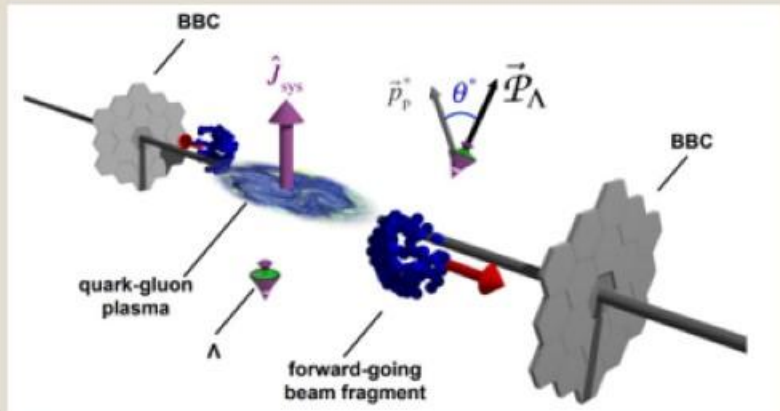
QCD Phase structure



The conjectured QCD phase diagram. CEP would directly determine the QCD phase structure.

Strong coupling non-perturbative approaches are required to study the phase transition region, and holographic QCD is applicable.

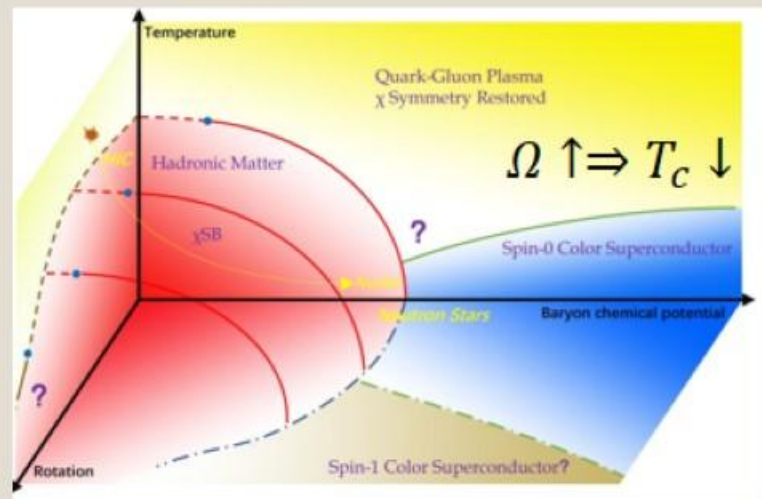
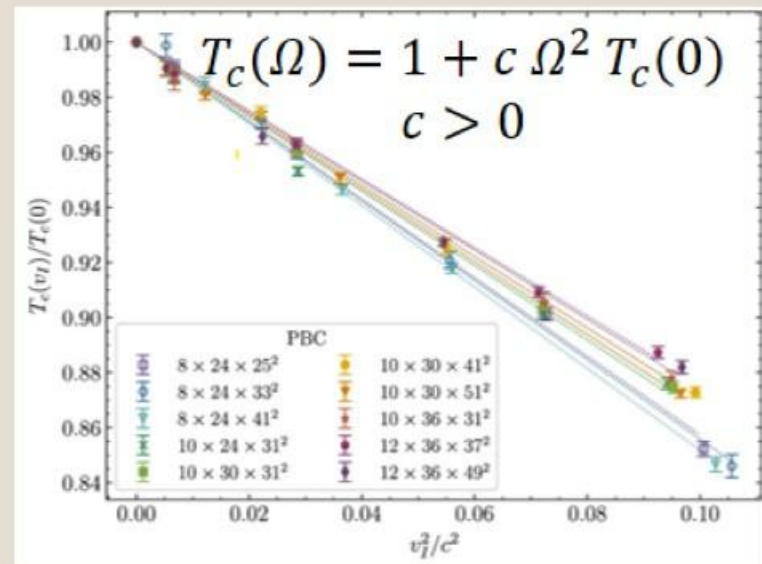
Rotation in heavy ion collisions



$$\omega \approx (9 \pm 1) \times 10^{21} \text{ s}^{-1}$$

Lattice QCD and effective models predict opposite temperature trends for QCD phase transition under rotation. How can we determine which one is physical?

Large angular velocity is generated in non-central heavy ion collisions. Whether such huge rotation affects experimental observables and the phase transitions of QCD matter.



STAR, Nature 548 (2017) 62-65.

Becattini, Liao, Lisa, Lect.Notes Phys. 987 (2021).

Braguta, Kotov, Kuznedelev, Roenko, Phys.Rev.D 103 (2021) 9, 094515.

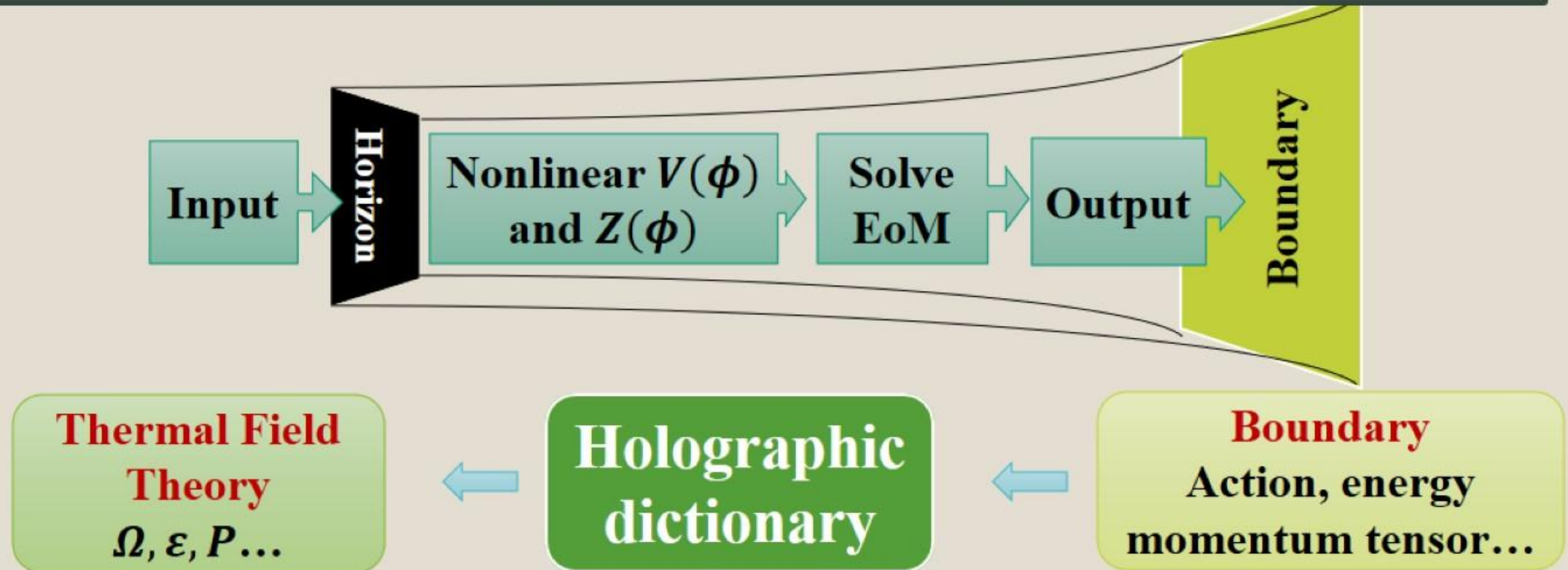
Wang, Chen, Hou, Ren, arXiv: 2505.15487.

Fukushima, Shimada, Phys.Lett.B 868 (2025) 139716.

Holographic renormalization

Gubser, Klebanov, Polyakov, Phys.Lett.B 428 (1998) 105-114.
Witten, Adv.Theor.Math.Phys. 2 (1998) 253-291.

Method for investigating strongly coupled dual field theory via weakly coupled gravity.



$$Z_{\text{bulk}}[\phi_0] \equiv \int_{\phi|_{\partial}=\phi_0} \mathcal{D}\phi e^{-S[\phi]} = \left\langle \exp\left(\int_{\partial\text{AdS}} d^d x \phi_0(x) \mathcal{O}(x)\right) \right\rangle_{\text{CFT}} \equiv W_{\text{CFT}}[\phi_0]$$

Gubser–Klebanov–Polyakov–Witten relation

Holographic renormalization

Einstein-Maxwell-Dilaton action

$$S = \frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) \right]$$

dilaton with conformal dim 3. Metric take the following form

$$ds^2 = -e^{-w(r)} f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\vec{x}^2$$

Assume dilaton only depend on holographic direction and the gauge field fixed as

$$\phi = \phi(r), \quad \text{and} \quad A_\mu dx^\mu = A_t(r) dt,$$

Holographic renormalization

ZL, Wang, arXiv: 2507.09113

EoM

$$w' + \frac{1}{3}r\phi'^2 = 0,$$

$$\phi'' + \left(\frac{3}{r} + \frac{f'}{f} - \frac{w'}{2}\right)\phi' + \frac{1}{2f}Z'(\phi)e^w A_t'^2 - \frac{1}{f}V'(\phi) = 0,$$

$$\partial_r [e^{w/2}r^3 Z(\phi)A_t'] = 0,$$

$$\frac{4}{r^2} + \frac{2V(\phi)}{3f} + \frac{Z(\phi)e^w A_t'^2}{3f} + \frac{2f'}{rf} - \frac{w'}{r} = 0.$$

$$f|_{r \rightarrow \infty} = r^2 + \frac{\phi_s^2}{6} - \frac{2\phi_s^3 V^{(3)}(0)}{9r} + \frac{f_v}{r^2} + \frac{\phi_s^4}{24r^2} [3V^{(3)}(0)^2 - V^{(4)}(0) - 2] \ln(r) + \mathcal{O}(r^{-3})$$

$$w|_{r \rightarrow \infty} = \frac{\phi_s^2}{gr^2} - \frac{2\phi_s^3 V^{(3)}(0)}{9r^3} + \mathcal{O}(r^{-4}),$$

$$\phi|_{r \rightarrow \infty} = \frac{\phi_s}{r} - \frac{\phi_s^2 V^{(3)}(0)}{2r^2} + \frac{\phi_v}{r^3} + \frac{\phi_s^3}{12r^3} [3V^{(3)}(0)^2 - V^{(4)}(0) - 2] \ln(r) + \mathcal{O}(r^{-4}),$$

$$A_t|_{r \rightarrow \infty} = \mu_B - \frac{a_v}{2r^2} + \frac{a_v \phi_s Z'(0)}{3Z(0)r^3} + \mathcal{O}(r^{-4}),$$

UV expansions

$$S_\partial = \frac{1}{2\kappa_5^2} \int d^4x \sqrt{-h} \mathcal{L}_\partial$$

Boundary term

$$= \frac{1}{2\kappa_5^2} \int d^4x \sqrt{-h} \left\{ 2K - 6 - \frac{1}{2}\phi^2 + \frac{1}{6}V^{(3)}(0)\phi^3 - b\phi^4 + \frac{1}{24} [2 - 3V^{(3)}(0)^2 + V^{(4)}(0)] \phi^4 \ln(r) \right\} \Big|_{r=\infty}$$

$$T_{ab} = \frac{2}{\sqrt{-\zeta}} \frac{\delta(S + S_\partial)_{on-shell}}{\delta \zeta^{ab}} = \lim_{r \rightarrow \infty} \left(\frac{2r^2}{\sqrt{-h}} \frac{\delta(S + S_\partial)_{on-shell}}{\delta h^{ab}} \right) = \frac{1}{2\kappa_5^2} \lim_{r \rightarrow \infty} r^2 \{ -2K_{ab} - Z(0)F_{ac}F_b{}^c \ln(r) + h_{ab} \mathcal{L}_\partial \}.$$

Energy momentum tensor on the boundary

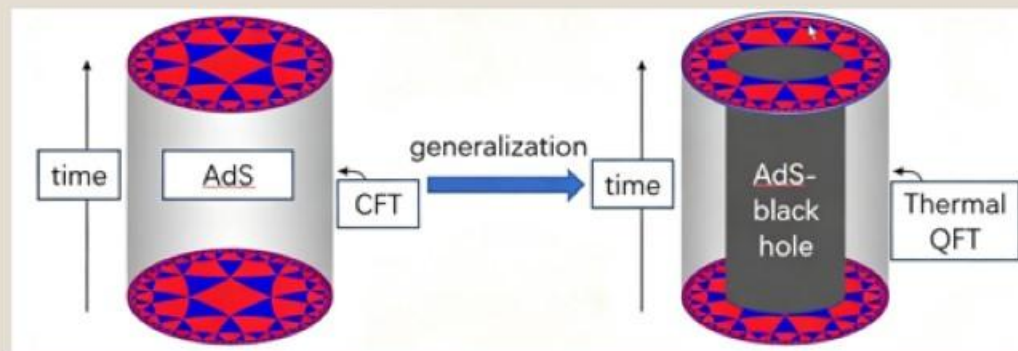
$$\frac{T}{\tilde{V}} (S + S_\partial)_{on-shell} = f_v - \phi_s \phi_v - \frac{\phi_s^4}{144} [9 - 144b - 12V^{(3)}(0)^2 + 2V^{(4)}(0)]$$

Renormalized on shell action

Holographic renormalization

ZL, Wang, arXiv: 2507.09113

$$\begin{aligned}\mathcal{E} &= T_{00} = \frac{1}{2\kappa_5^2} \left\{ -3f_v + \phi_s \phi_v + \phi_s^4 \left[\frac{1}{48} + b + \frac{3}{8} V^{(3)}(0)^2 \right] \right\}, \\ P_x &= P_y = P_z = T_{ii} \\ &= \frac{1}{2\kappa_5^2} \left\{ -f_v + \phi_s \phi_v + \frac{\phi_s^4}{144} [9 - 144b - 12V^{(3)}(0)^2 + 2V^{(4)}(0)] \right\} \\ &= -\frac{T}{\tilde{V}} (S + S_\partial)_{on-shell}, \\ I &= \mathcal{E} - 3P = -\frac{1}{2\kappa_5^2} \left\{ 2\phi_s \phi_v + \frac{\phi_s^4}{24} [4 - 96b - 15V^{(3)}(0)^2 + V^{(4)}(0)] \right\}.\end{aligned}$$



$$\delta P = \delta (-\mathcal{E} + \mu_B \rho_B + T s) = \rho_B \delta \mu_B + s \delta T$$

Thermodynamic relations of holographic models agree with the grand canonical ensemble.

$$V(\phi) =$$

$$Z(\phi) =$$



Physics of QCD

Appropriate choices of Z and V yield a reasonable description of the properties of QCD matter.

Pseudo-Lorentz boost method

Zhao, He, Hou, Li, **ZL**,10.1007/JHEP04(2023)115.

$$S_M = \frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g} [\mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi)]$$

$$t \rightarrow \frac{1}{\sqrt{1 - \omega^2 \ell^2}} (\hat{t} + \omega \ell^2 \hat{\theta}), \quad \theta \rightarrow \frac{1}{\sqrt{1 - \omega^2 \ell^2}} (\hat{\theta} + \omega \hat{t}).$$

$$d\hat{s}^2 = g_{\mu\nu} d\hat{x}^\mu d\hat{x}^\nu = -N(r) d\hat{t}^2 + \frac{dr^2}{f(r)} + R(r) (d\hat{\theta} + Q(r) d\hat{t})^2 + r^2 d\sigma^2,$$

$$N(r) = \frac{r^2 f(r) (1 - \omega^2 \ell^2)}{r^2 e^{\eta(r)} - \omega^2 \ell^2 f(r)},$$

$$R(r) = \frac{r^2 \ell^2 - \omega^2 \ell^4 f(r) e^{-\eta(r)}}{1 - \omega^2 \ell^2},$$

$$Q(r) = \frac{\omega (f(r) - r^2 e^{\eta(r)})}{\omega^2 \ell^2 f(r) - r^2 e^{\eta(r)}}.$$

$$V(\phi) = -12 \cosh [c_1 \phi] + \left(6c_1^2 - \frac{3}{2} \right) \phi^2 + c_2 \phi^6,$$

$$Z(\phi) = \frac{1}{1 + c_3} \text{sech}[c_4 \phi^3] + \frac{c_3}{1 + c_3} e^{-c_5 \phi}.$$

model	c_1	c_2	c_3	c_4	c_5	κ_N^2	$\phi_s(\text{GeV})$	b
pure $SU(3)$	0.735	0				$2\pi(4.88)$	1.523	-0.36458
2+1 flavor	0.710	0.0037	1.935	0.085	30	$2\pi(1.68)$	1.085	-0.27341

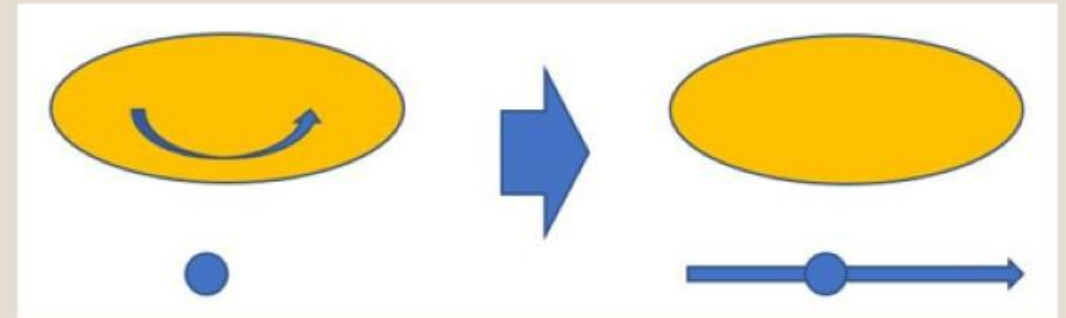
$$\hat{\epsilon} := \hat{T}_{00} = \frac{\epsilon + \omega^2 \ell^2 P}{1 - \omega^2 \ell^2},$$

$$\hat{P}_1 := \hat{T}_{11} = \frac{1}{\ell^2} \hat{T}_{\theta\theta} = \frac{P + \omega^2 \ell^2 \epsilon}{1 - \omega^2 \ell^2},$$

$$\hat{P}_2 := \hat{T}_{22} = P,$$

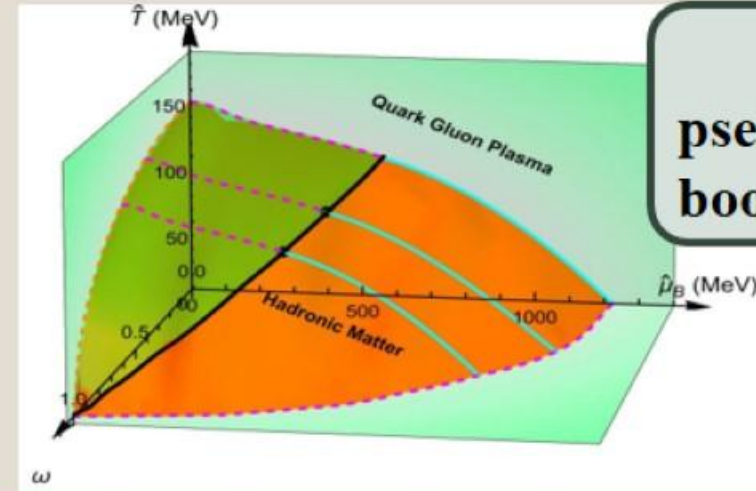
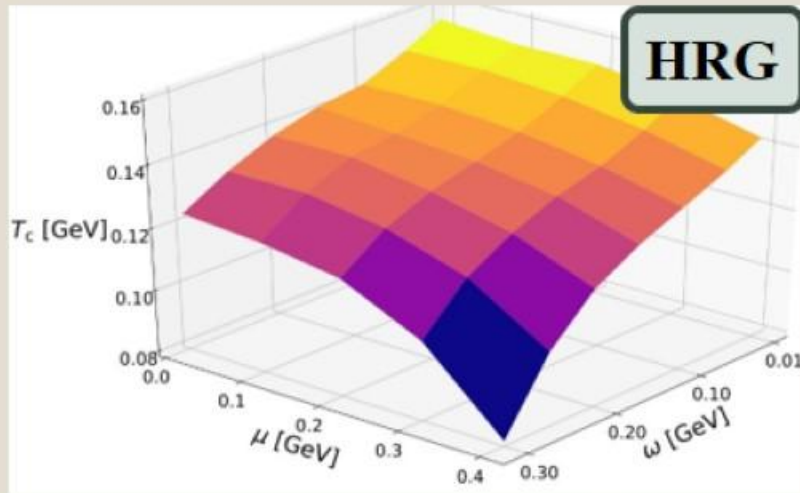
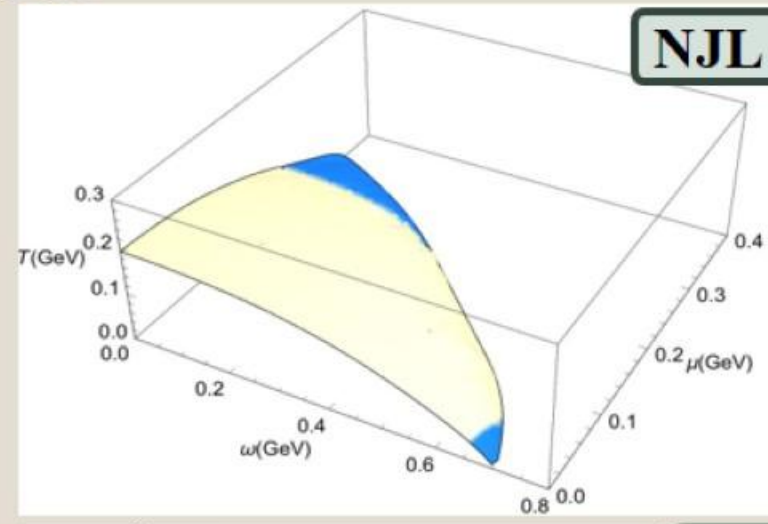
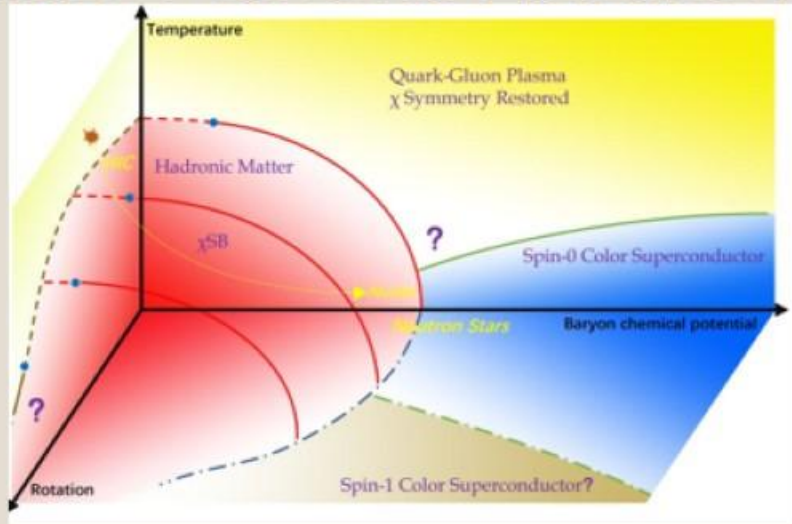
$$\hat{P}_3 := \hat{T}_{33} = P,$$

$$\hat{T}_{\theta t} = \frac{\omega \ell^2 (\epsilon + P)}{1 - \omega^2 \ell^2},$$



**HQCD
pseudo-Lorentz
boost method**

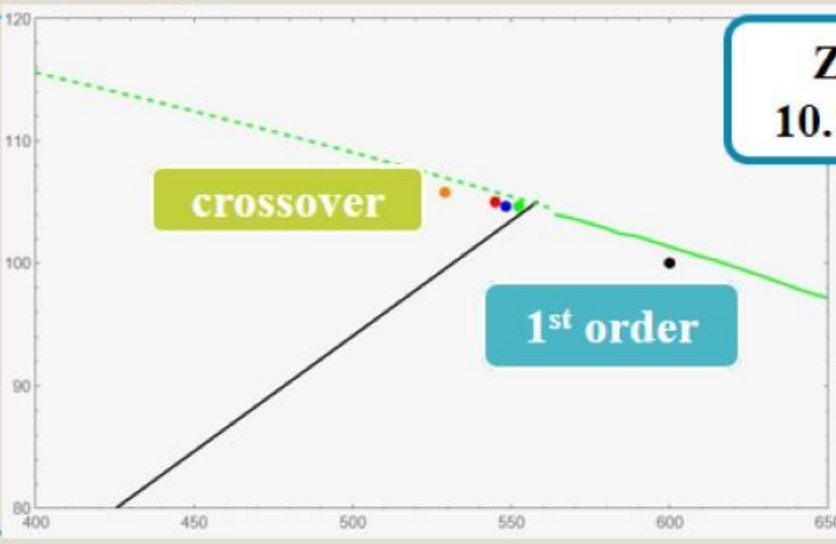
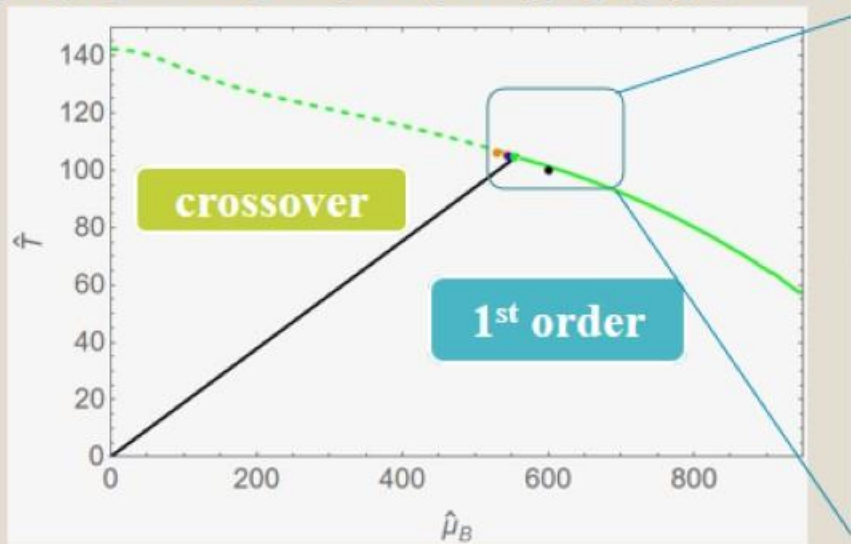
Pseudo-Lorentz boost method



Various effective models yield similar results:
the phase transition temperature decreases as
the angular velocity increases.

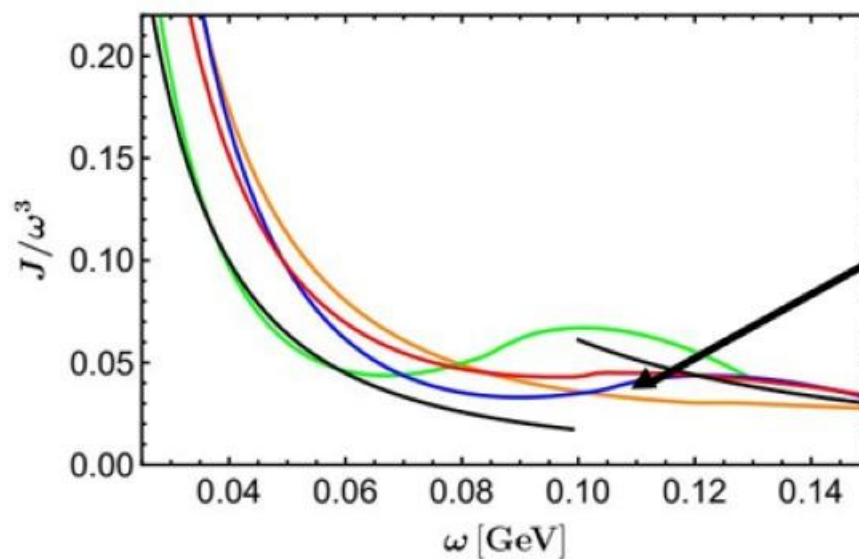
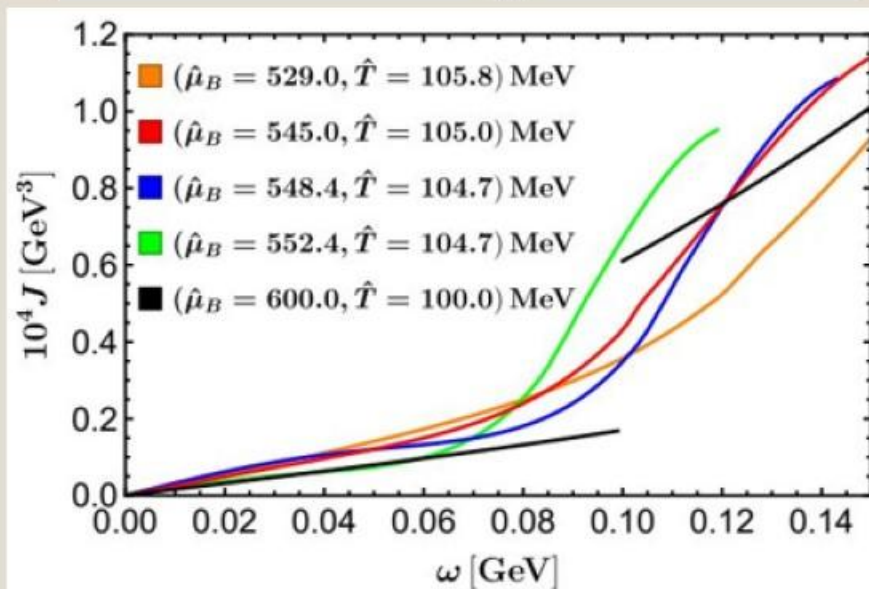
Becattini, Liao, Lisa, Lect. Notes Phys. 987 (2021).
Wang, Wei, **ZL**, Huang, Phys. Rev. D 99 (2019) 1, 016018.
Fujimoto, Fukushima, Hidaka, Phys. Lett. B 816 (2021) 136184.
Zhao, He, Hou, Li, **ZL**, 10.1007/JHEP04(2023)115.

Pseudo-Lorentz boost method



Zhao, He, Hou, Li, **ZL**,
10.1007/JHEP04(2023)115.

In crossover region, if near the CEP, J/ω^3 shows a peak structure.

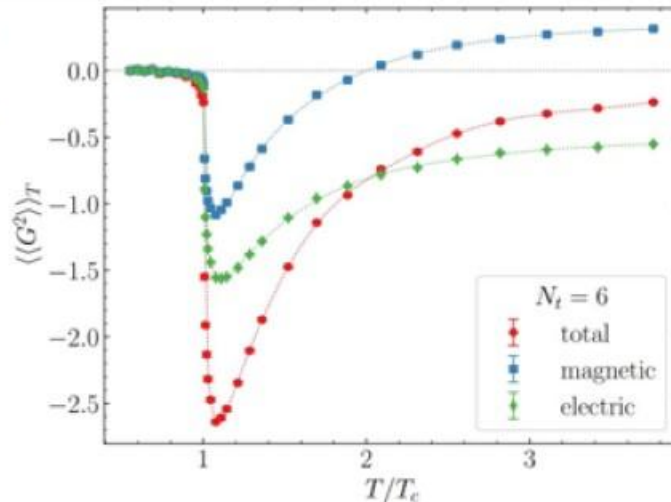
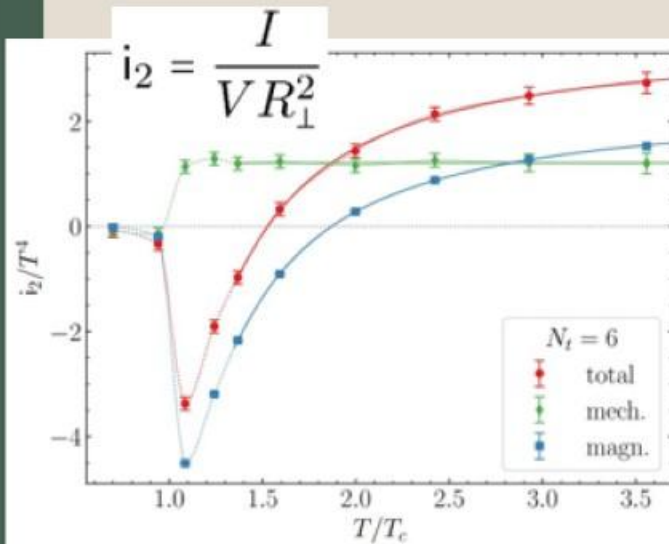
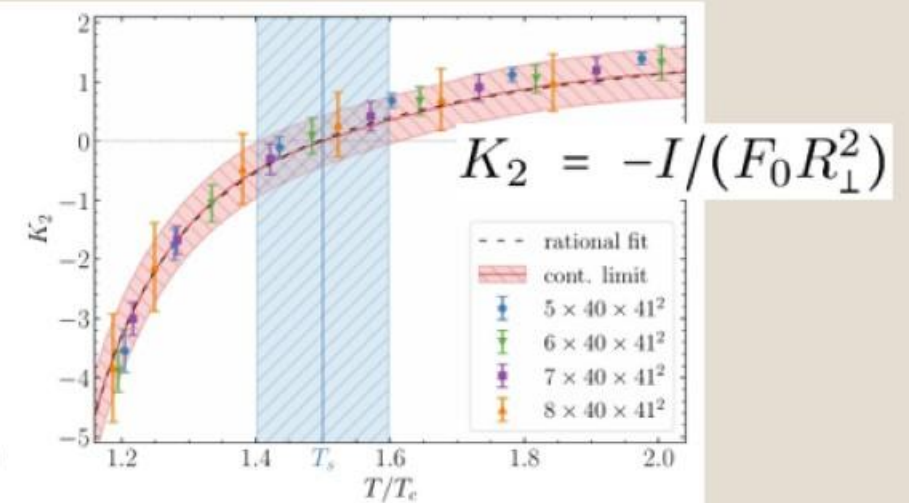
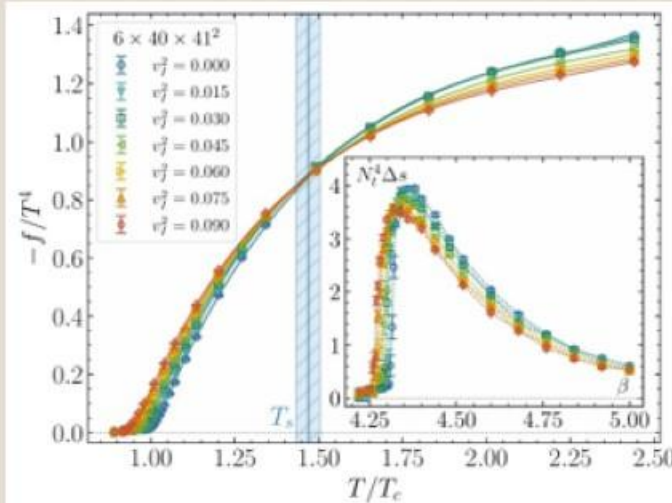
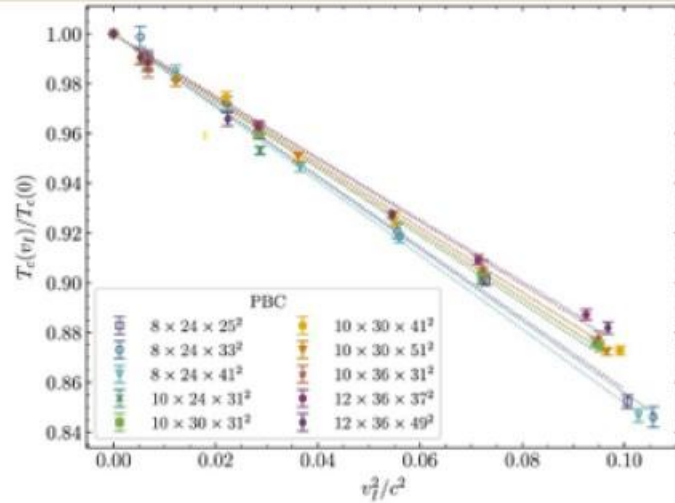


Possible
signal of
CEP

Behavior of angular momentum in HQCD.

Rotation–magnetic correspondence

Bragutaa, et al., Phys. Lett. B 852 (2024) 138604.
Braguta, et al., Phys.Rev.D 103 (2021) 9, 094515.



$$I^{\text{gl}} = I_{\text{mech}}^{\text{gl}} + I_{\text{magn}}^{\text{gl}}$$

$$I_{\text{magn}}^{\text{gl}} = \frac{2}{3} \int_V d^3x x_{\perp}^2 \langle\langle (B^a)^2 \rangle\rangle_T$$

**Variation of pressure under
lattice QCD imaginary
rotation and emergence of
negative moment of inertia.**

Rotation–magnetic correspondence

Nonrelativistic Lagrangian for charged particle q , mass m :

$$L_{\text{EM}} = T - V = \frac{1}{2}m\mathbf{v}^2 - q\phi + q\mathbf{A}_{\text{em}} \cdot \mathbf{v}$$

Lagrangian in rotating frame

$$L_{\text{rot}} = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\dot{\mathbf{r}} \cdot (\boldsymbol{\Omega} \times \mathbf{r}) + \frac{1}{2}m(\boldsymbol{\Omega} \times \mathbf{r})^2.$$

Charged particle in electromagnetic field

Electric force: $\mathbf{F}_E = q\mathbf{E}$

Magnetic Lorentz force: $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$

Electric field: $\mathbf{E} = -\nabla\phi$

Scalar potential: ϕ

Magnetic field: $\mathbf{B} = \nabla \times \mathbf{A}_{\text{em}}$

Vector potential: $\mathbf{A}_{\text{em}}$

Particle in uniformly rotating frame

Centrifugal inertial force: $\mathbf{F}_{\text{cf}} = m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$

Coriolis force: $\mathbf{F}_{\text{Cor}} = 2m\mathbf{v}' \times \boldsymbol{\Omega}$

Effective electric field: $\mathbf{E}_{\text{eff}} = -\frac{m}{q}\nabla\left(-\frac{1}{2}(\boldsymbol{\Omega} \times \mathbf{r})^2\right)$

Effective scalar potential: $\phi_{\text{eff}} = -\frac{m}{q} \cdot \frac{1}{2}(\boldsymbol{\Omega} \times \mathbf{r})^2$

Effective magnetic field: $\mathbf{B}_{\text{eff}} = \frac{m}{q} \cdot 2\boldsymbol{\Omega}$

Effective vector potential: $\mathbf{A} = \frac{m}{q}(\boldsymbol{\Omega} \times \mathbf{r})$

Lagrangian for charged particle in electromagnetic field

$$L_{\text{EM}} = \frac{1}{2}m\dot{\mathbf{r}}^2 - q\phi + q\mathbf{A}_{\text{em}} \cdot \dot{\mathbf{r}}$$

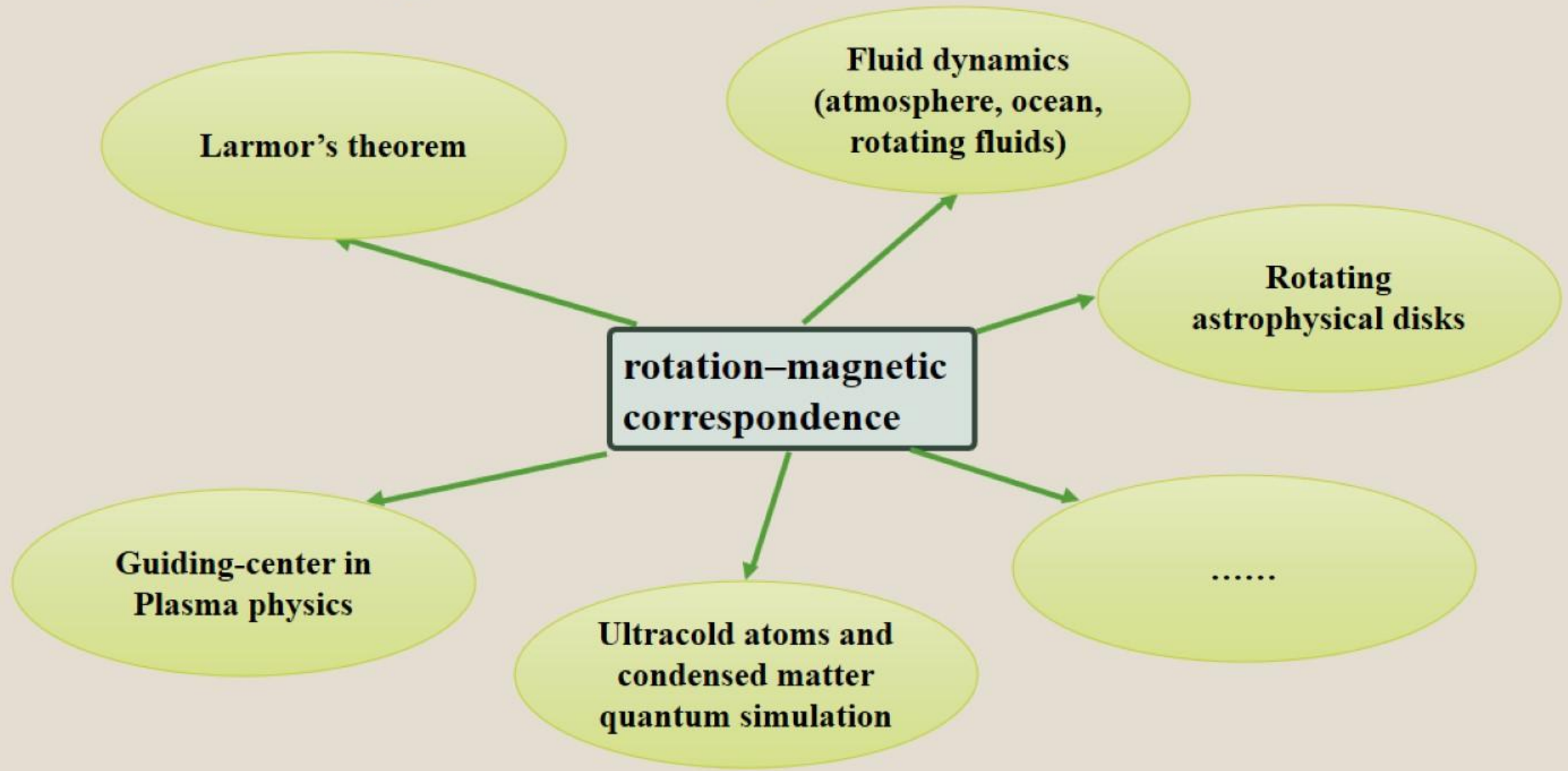
Lagrangian in uniformly rotating frame (effective EM analogy)

$$L_{\text{rot}} = \frac{1}{2}m\dot{\mathbf{r}}'^2 - q\phi_{\text{eff}} + q\mathbf{A}_{\text{eff}} \cdot \dot{\mathbf{r}}'$$

$$\mathbf{B} = 2m(T)\boldsymbol{\Omega}$$

The motion of a charged particle in electromagnetic fields and particle motion in a rotating frame share mathematical isomorphism.

Rotation–magnetic correspondence



Rotation–magnetic correspondence

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) - \frac{Z(\phi)}{4} F^2 \right]$$

$$V(\phi) = -12 \cosh(\gamma\phi) + \left(6\gamma^2 - \frac{3}{2}\right) \phi^2$$

$$Z(\phi) = \text{sech}(\zeta\phi^3) \quad \zeta = 0.1275$$

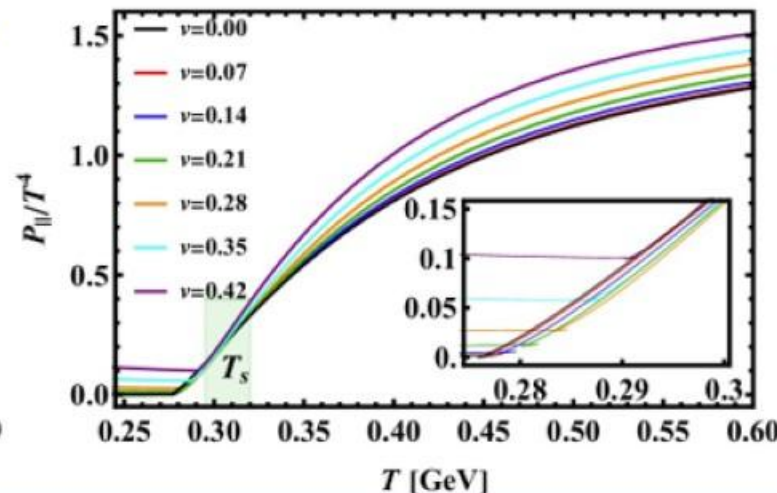
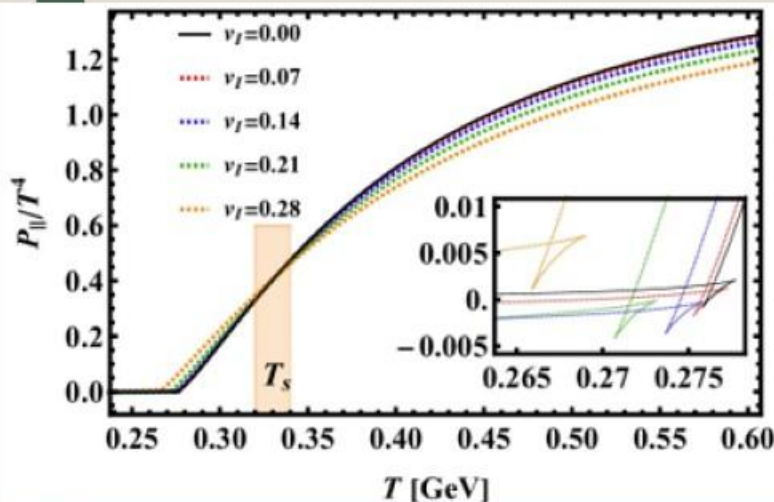
$$\gamma = 0.735, \quad \kappa_5^2 = 9.76\pi, \quad \phi_s = 1.523 \text{ GeV}$$

ZL, Chen, Huang, arXiv: 2607.23199.

$$B = 2m(T)\Omega$$

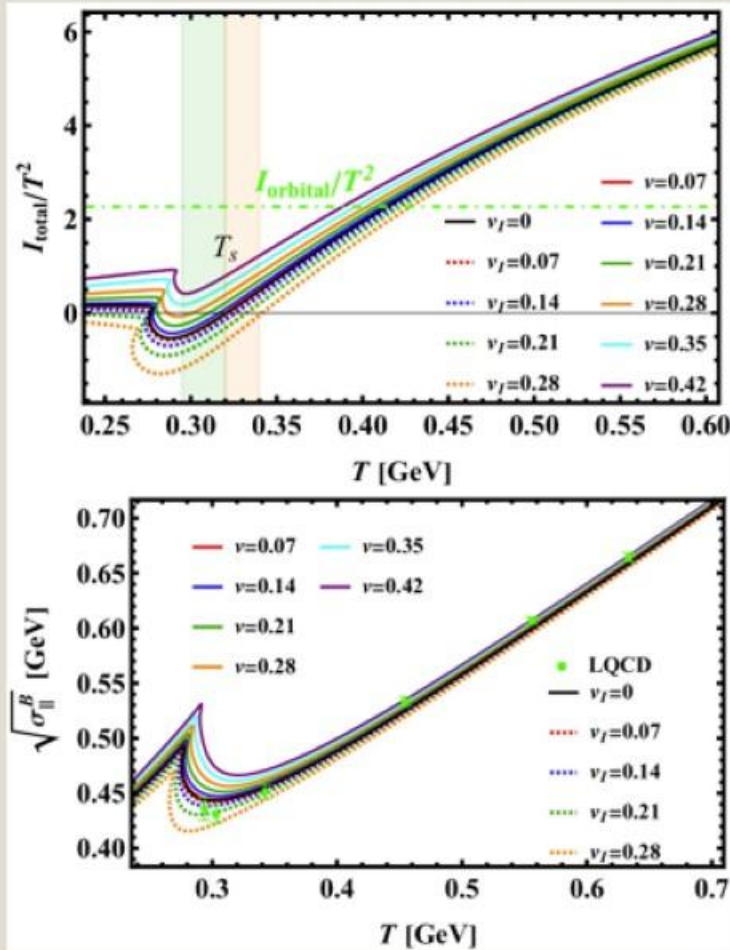
$$m(T) = \lambda T$$

The gluon mass is taken proportional to temperature, following its Debye mass.



Reproduction of lattice imaginary rotation and its analytic continuation to real rotation.

Rotation–magnetic correspondence

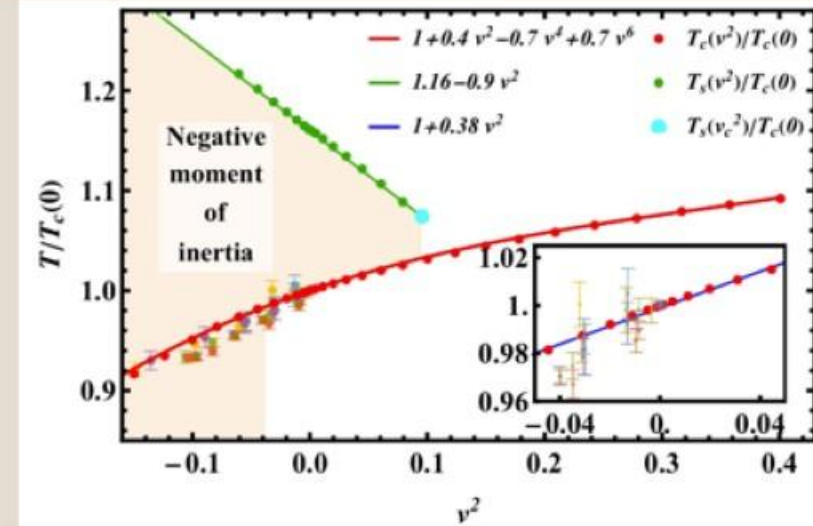


The moment of inertia and chromomagnetic string tension exhibit highly similar behaviors.

Boyd, et al., Nucl. Phys. B 469, 419 (1996).
 Braguta, et al., Phys.Rev.D 103 (2021) 9, 094515.
 ZL, Chen, Huang, arXiv: 2607.23199.

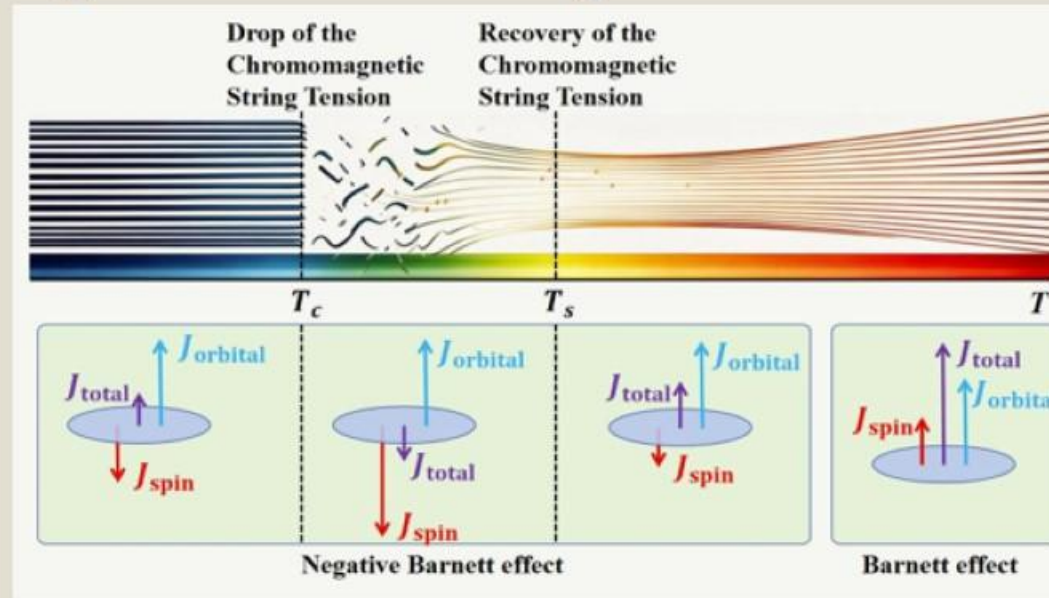
$$P_{\parallel} - P_{\perp} = \frac{1}{2\kappa_5^2} \left[m^2 \Omega^2 Z(0) + \frac{4g_v}{g_0} \right] = \Omega J_{\text{total}}$$

$$I_{\text{orbital}} = \frac{1}{2\kappa_5^2} m^2 Z(0) \quad I_{\text{spin}} = \frac{1}{2\kappa_5^2} \frac{4g_v}{g_0 \Omega^2}$$



The phase-transition temperature varies with angular velocity.

Rotation–magnetic correspondence



ZL, Chen, Huang, arXiv: 2607.23199.

Chromomagnetic-induced inertia inversion (CII) mechanism

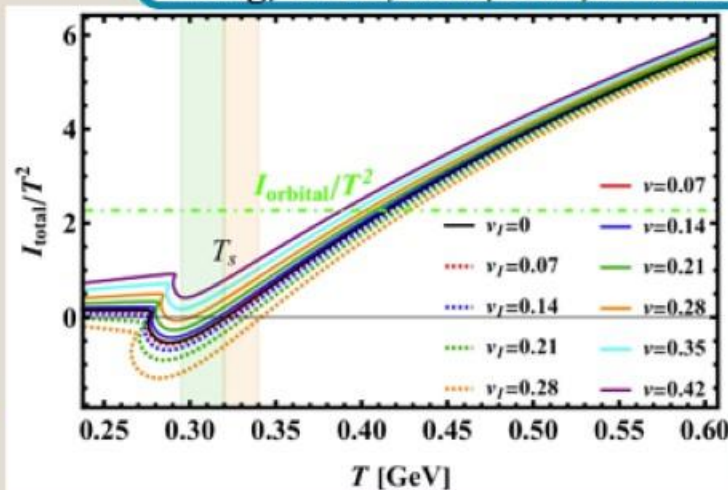
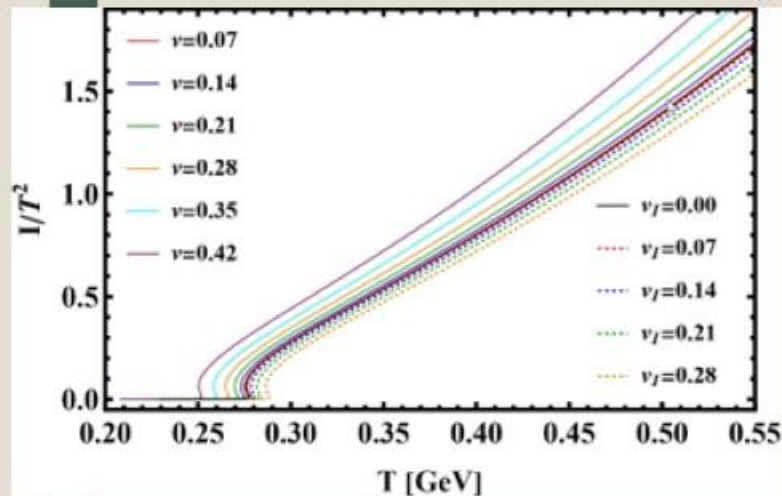
- Real (imaginary) rotation increases (reduces) the chromomagnetic string tension, thereby raising (lowering) the phase transition temperature.
- At the phase transition, the chromomagnetic string tension drops sharply, triggering a rapid fall of the moment of inertia that may turn negative, giving rise to the negative Barnett effect.
- At high temperatures, the chromomagnetic string tension is restored, the moment of inertia returns to positive values, and the conventional Barnett effect emerges.

Combined analysis

Zhao, He, Hou, Li, **ZL**, 10.1007/JHEP04(2023)115.

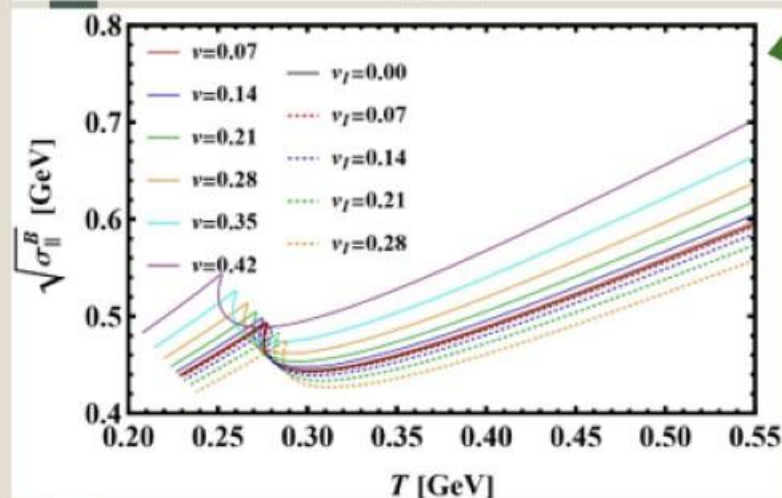
ZL, Chen, Huang, arXiv: 2607.23199.

Wang, Chen, Hou, Ren, arXiv: 2505.15487.

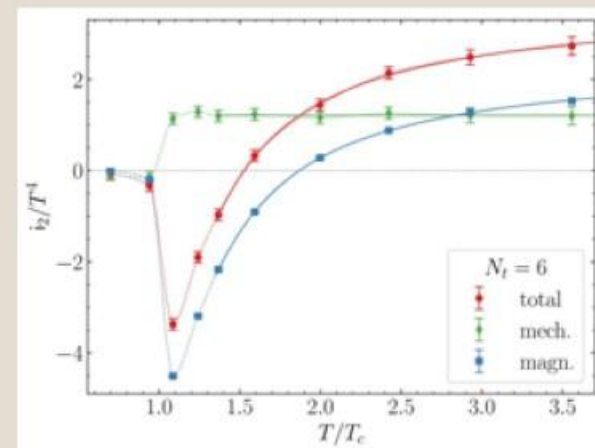
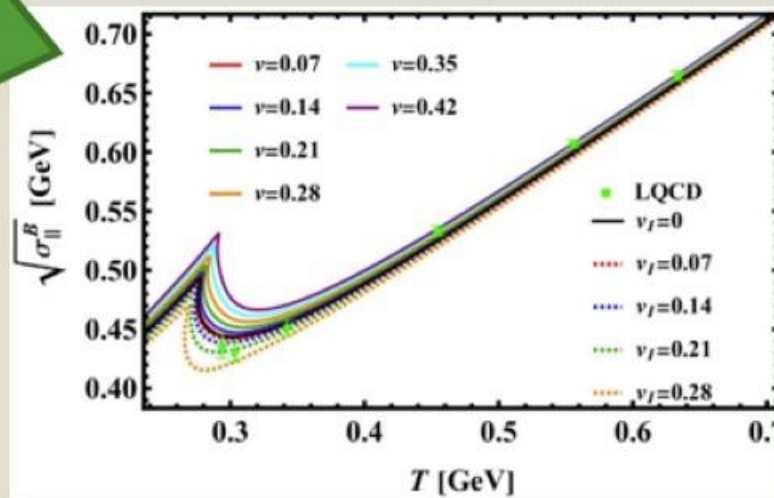


$$T_d = T_d^{(0)} (1 + Bv^2)$$

$$B = -\frac{\Delta I}{2R^2 L}$$



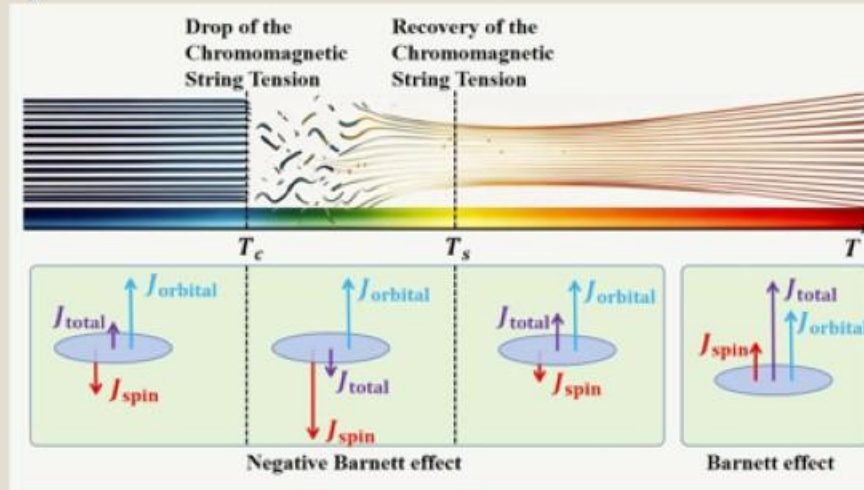
VS



Just above T_c chromomagnetic string tension diminishes with angular velocity, possibly reducing T_c .

$$I_{\text{magn}}^{\text{gl}} = \frac{2}{3} \int_V d^3x x_{\perp}^2 \langle\langle (B^a)^2 \rangle\rangle_T$$

Combined analysis

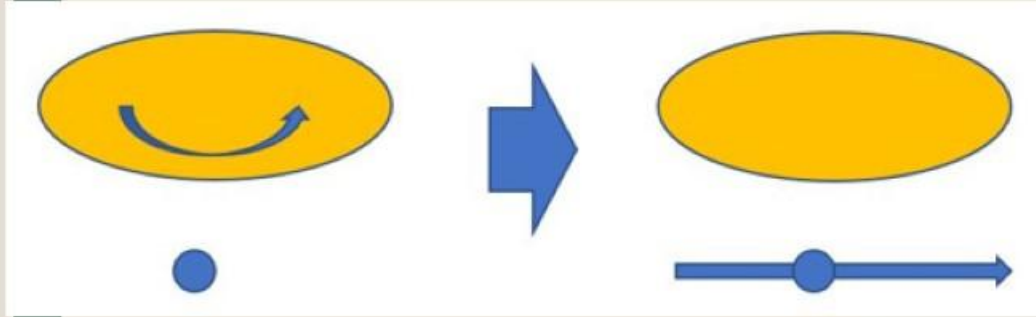


ZL, Chen, Huang, arXiv: 2607.23199.

Chromomagnetic-induced inertia inversion (CII) mechanism

- Outside the critical regime, rotation enhances the chromomagnetic string tension.
- Rotation increases (reduces) the chromomagnetic string tension slightly above T_c , thereby raising (lowering) the phase transition temperature.
- At the phase transition, the chromomagnetic string tension drops sharply, triggering a rapid fall of the moment of inertia that may turn negative, giving rise to the negative Barnett effect.
- At high temperatures, the chromomagnetic string tension is restored, the moment of inertia returns to positive values, and the conventional Barnett effect emerges.

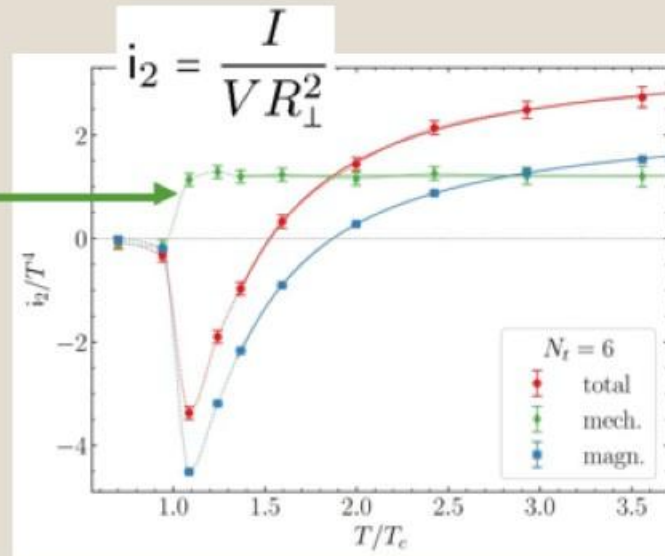
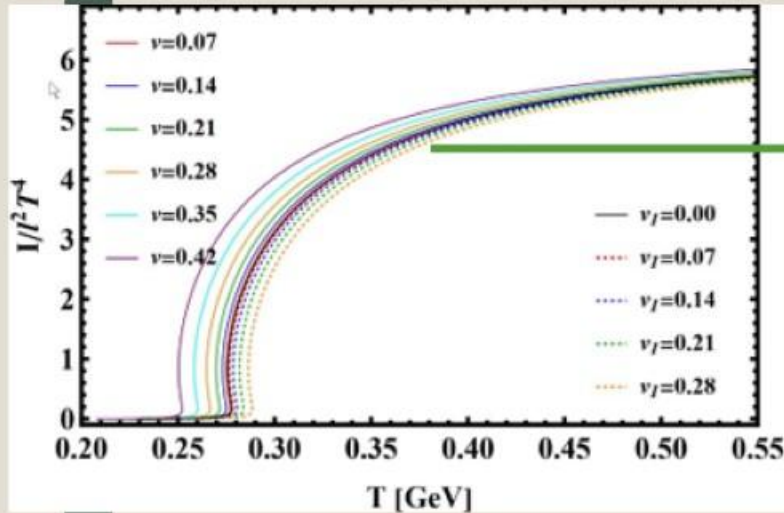
Combined analysis



$$P_{\parallel} - P_{\perp} = \frac{1}{2\kappa_5^2} \left[m^2 \Omega^2 Z(0) + \frac{4g_v}{g_0} \right] = \Omega J_{\text{total}}$$

$$I_{\text{orbital}} = \frac{1}{2\kappa_5^2} m^2 Z(0) \quad I_{\text{spin}} = \frac{1}{2\kappa_5^2} \frac{4g_v}{g_0 \Omega^2}$$

Though arising from kinematics, the rotation-magnetic field correspondence naturally induces dynamical responses.

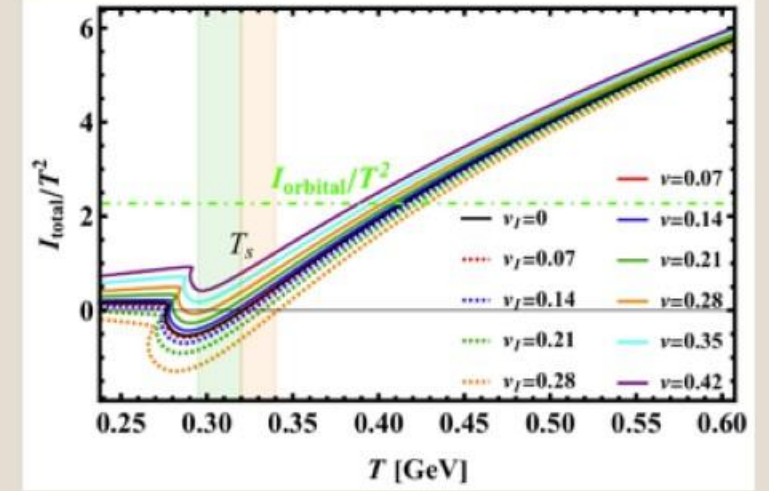
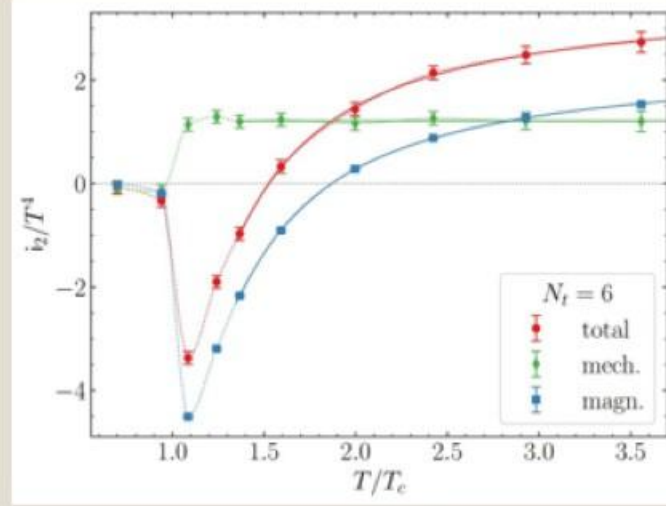
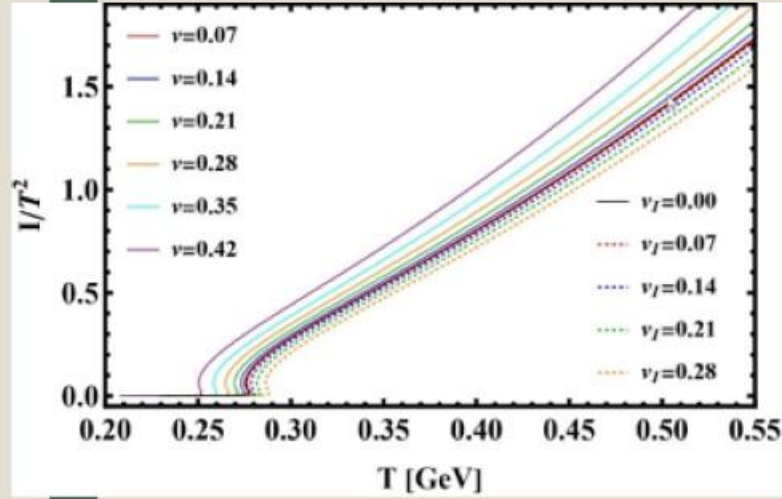


$$I_{\text{magn}}^{\text{gl}} = \frac{2}{3} \int_V d^3x x_{\perp}^2 \langle\langle (B^a)^2 \rangle\rangle_T$$

Only kinematic contributions are included in the pseudo-Lorentz rotation method?

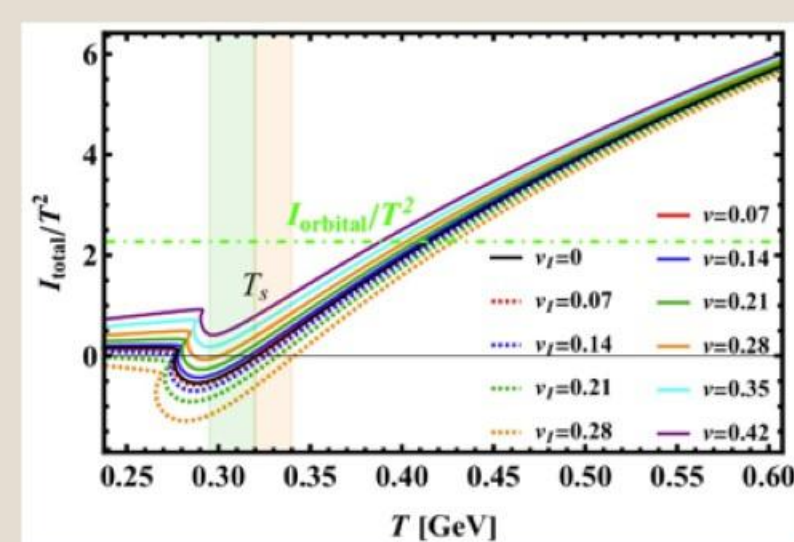
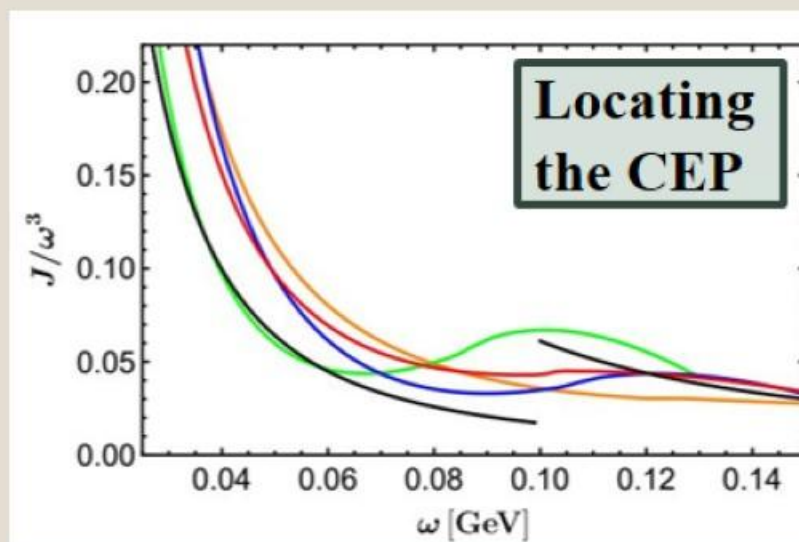
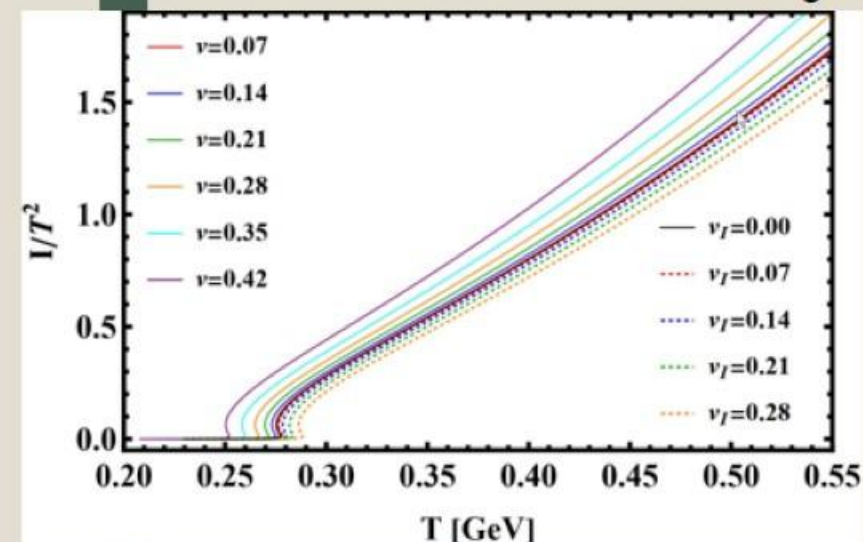
A possible clue: all physical quantities scale with multiples of the Lorentz factor: $\sqrt{1 - \omega^2 l^2}$.

Combined analysis



When we consider rotating QCD matter, we must account for its **intrinsic dynamical response** to rotation, particularly the dynamical response of its chromomagnetic sector. Conclusions from universal field theory or generic effective models may therefore be incomplete.

Combined analysis



Rapid increase of I slightly above T_c .

Peak structure of J/ω^3 .

Negative J or I .

Rapid drop of I slightly above T_c .

?

The phase transition temperature increases or decreases with increasing rotation..

$$B = -\frac{\Delta I}{2R^2 L}$$

Zhao, He, Hou, Li, [ZL](#), 10.1007/JHEP04(2023)115.
[ZL](#), Chen, Huang, arXiv: 2607.23199.
 Wang, Chen, Hou, Ren, arXiv: 2505.15487.

Conclusion & questions

1. Pseudo-Lorentz boost method yield a decreasing phase transition temperature with growing angular velocity, in contradiction to lattice QCD predictions.
 2. The rotation-magnetic correspondence approach yields rotation responses consistent with lattice QCD and motivates the CII mechanism.
 3. The sign of the change in moment of inertia across the phase transition governs whether the transition temperature increases or decreases.
 4. The dynamical contribution to the moment of inertia may be closely related to the nonperturbative chromomagnetic sector.
- I. Can experiments tell whether the phase transition temperature increases or decreases with rotation?
 - II. How does the nonperturbative dynamics of QCD matter affect rotational response?
 - III.

Thanks for your attention!